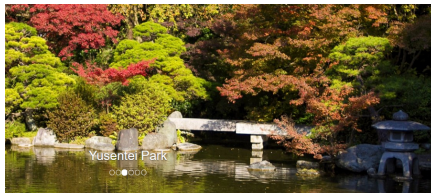


A General Methodology for Internalising Multi-Level Model Typing

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University of Applied Sciences, FHDW Hannover, Germany
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University of Bergen, Norway

October 12, 2021



MULTI 2021

Contributions

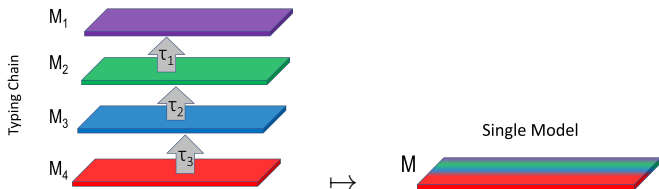
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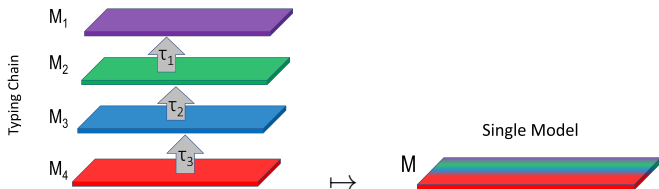
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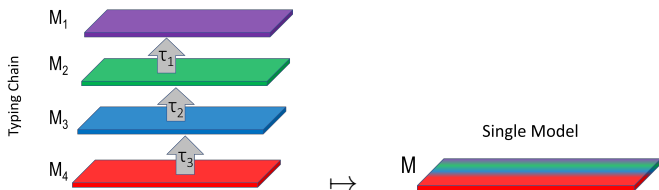


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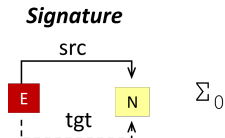
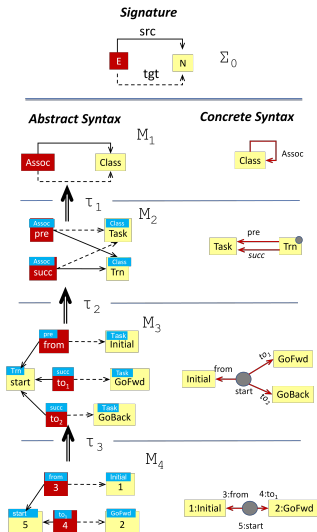
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Benefits: Avoid complicated multi-level handling (e.g. model transformations, co-evolution). Current limitations: No partial typings (potency > 1), no tooling so far.

Example: Models as Graphs (with reified edges)



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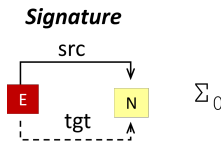
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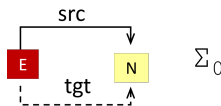
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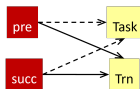
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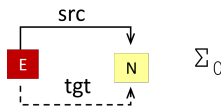
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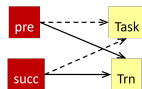
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$$E = \{pre, succ\}, N = \{Task, Trn\}$$

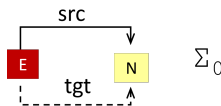
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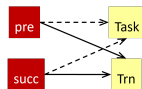
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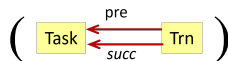
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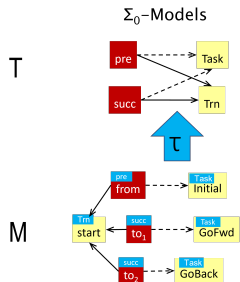
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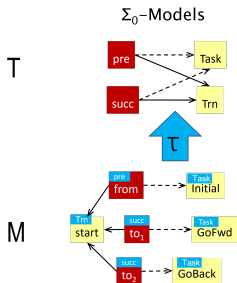
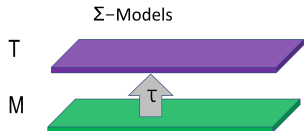
Internalising Model Typings: Atomic Step



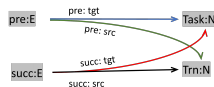
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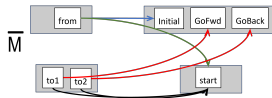
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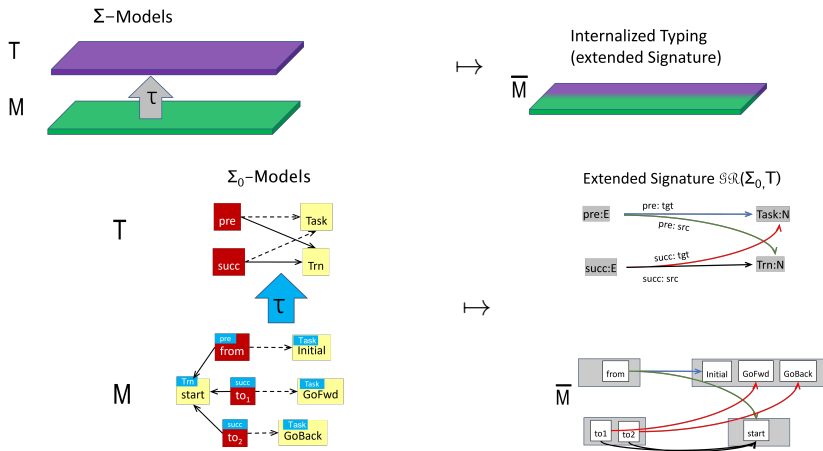
Extended Signature $\mathcal{SR}(\Sigma_0, T)$



\mapsto

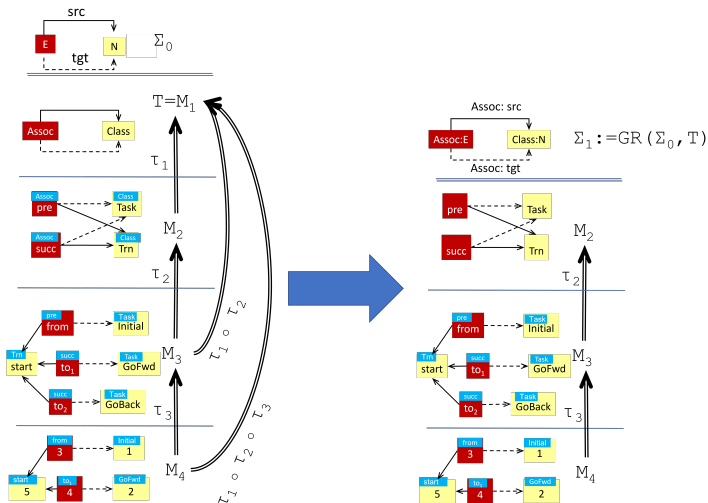


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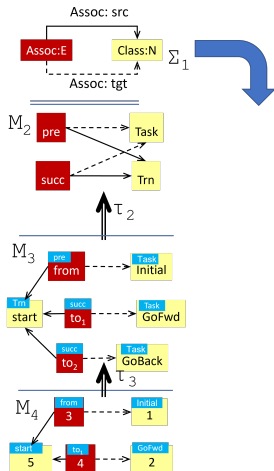


Based on categorical theorem: $\text{MOD}(\Sigma) \downarrow T \cong \text{MOD}(\mathcal{SR}(\Sigma, T))$.

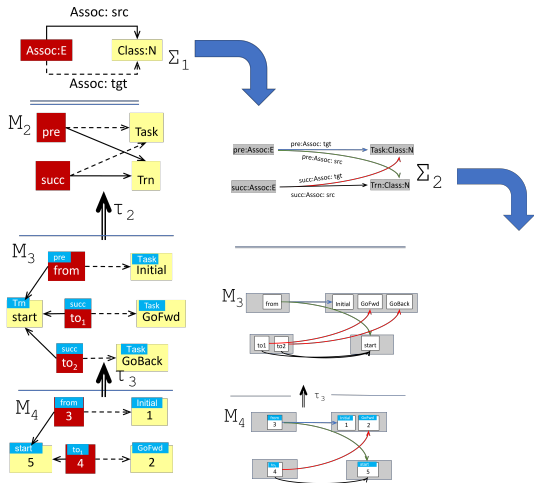
Comprehensive Application of Atomic Step



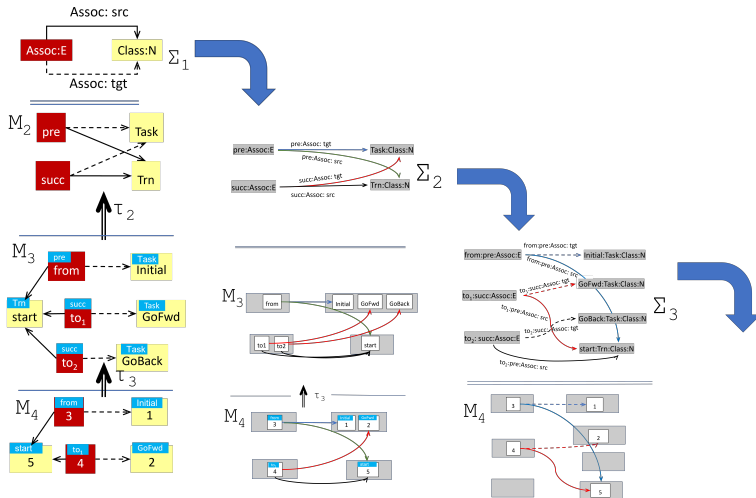
Iterating



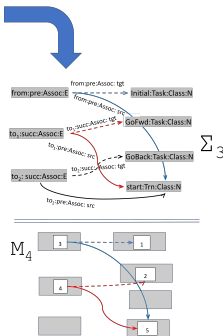
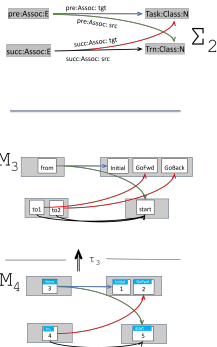
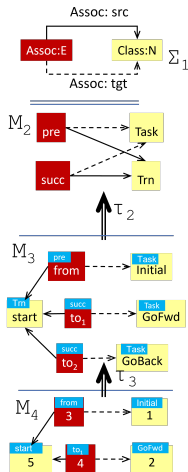
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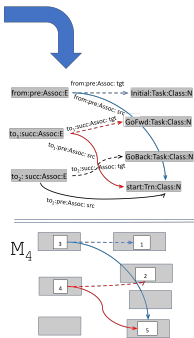
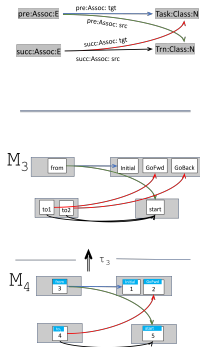
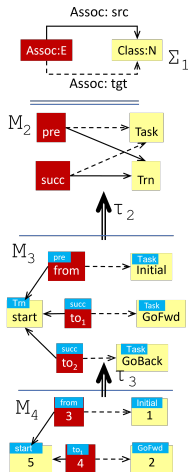


Last step:

$$\tau_n := id_{M_n}$$

Final result Σ_n

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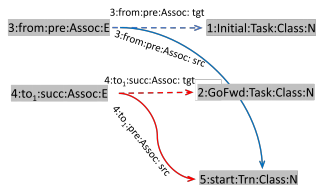
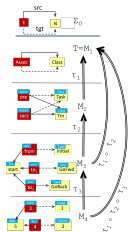


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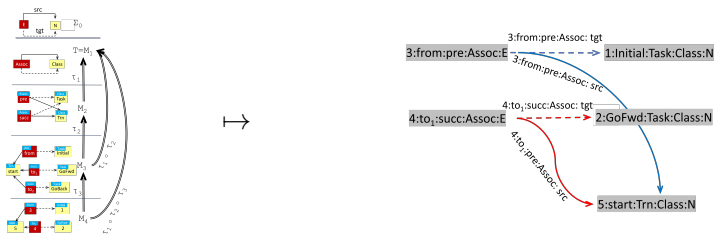
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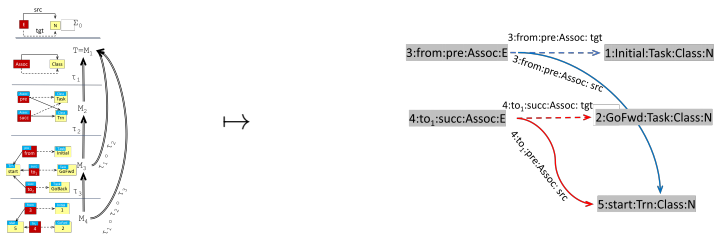


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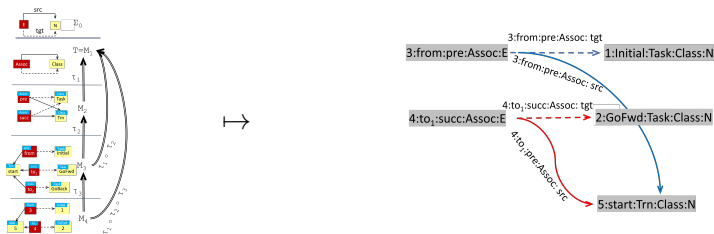
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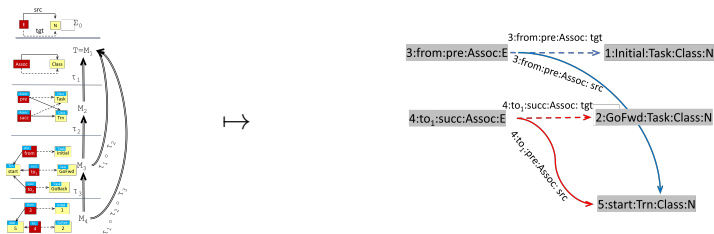
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